be complete cancellation (the well-ordered phase reversals of our two examples will not occur), but we can reasonably expect the level will be low. The reason is that the large phase jump from unit to unit distributes the phase angle of each unit's contribution "randomly." Thus the net contribution is kept low.

If we raise the frequency to the point where there is exactly 1 wavelength spacing between tweeters, their vectors will all come back into phase. Relative to the nearest element, each subsequent element's vector will have one full cycle of additional phase shift. Thus all units will add in phase and (for the far-field model) the level at 90° will be just as strong as that at 0° [2].

The unit-to-unit path length difference is at its maximum when the array is viewed at 90°. For angles less than this, the *observed* interunit spacing compresses by the sine of the angle. When viewed at 60°, the observed interunit spacing falls to 87% of the right-angle spacing. Our scenario for exactly 1 wavelength spacing will still occur, but at 1.16 times the frequency (1/0.866). Thus as the frequency rises, our lobe bends inward (4970 Hz, Fig. 13).

Clearly at twice the frequency of the first lobe, exactly 2 wavelengths will fit into the interunit spacing and a second sidelobe will occur (8600 Hz, Fig. 14).

2 RELATIONSHIP BETWEEN FOURIER TRANSFORM AND POLAR RESPONSE

Of course, rather than exciting our array with sine waves, we could impulse excite it. The Fourier transform of the received impulse response would then give the frequency response of the array at that angle. By inspection we can state that at any vertical angle other than on axis, the impulse fed to an array of elements is spread into a pulse train. The interpulse spacing is equal to the interunit time delay of the elements as observed from that vantage point (Fig. 16).

For any given frequency, the polar response equals the magnitude of the Fourier transform (at that frequency) evaluated over the varying pulse traing spacing created by the array's rotation. Furthermore, varying the array angle and, thus, varying the interpulse spacing is in effect the same as varying the applied wavelength. *The Fourier transform for all frequencies can be related to the polar response for a fixed frequency.*

The author accepts that the last statement is a bit of a stretch, but will offer an example as partial proof.

2.1 Mapping the Fourier Transform into a Polar Response

Fig. 17 shows the discrete Fourier transform (DFT) of a simple five-impulse sequence as might be observed off axis from a five-tweeter array. It is created by plotting the DFT of a 128-point sequence of five samples of magnitude 1.0 and 123 samples of magnitude 0.0. This "zero padding" is used purely to expand the resolution of the transform. The log magnitude plot of the 128-point transform can be seen to be a mirror function with symmetry around the Nyquist frequency.

Fig. 18 then shows the far-field polar response of a similarly configured five-element array. The array is defined as follows:

- Five elements, each of unity magnitude
- An element-to-element spacing of 80 mm
- 12.9-kHz excitation.

Note that from 0 through 90° the polar plot shows a triple repetition of a pattern of major and minor lobes. Between each pair of major lobes there are three minor lobes from 12 to 14 dB down in level. This pattern appears identical to the plot of the five-impulse DFT (Fig. 17).

Using the speed of sound of 344 ms/s we calculate that the 12.9-kHz wavelength is 0.02667 m. This was chosen to be exactly one-third the interunit spacing, so clearly when observing the array from the 90° angle, there will be an integral number of wavelengths between units (3 wavelengths). As a matter of interest, we can calculate the additional angles at which other integral numbers of wavelength can evenly fit into the observed interunit spacing using the following formula. For 2 wavelengths to fit,

$$\arcsin 2/3 = 41.81°$$

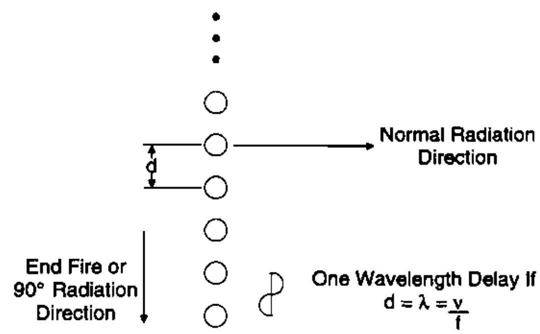

Fig. 15. Radiation 90° from normal axis.

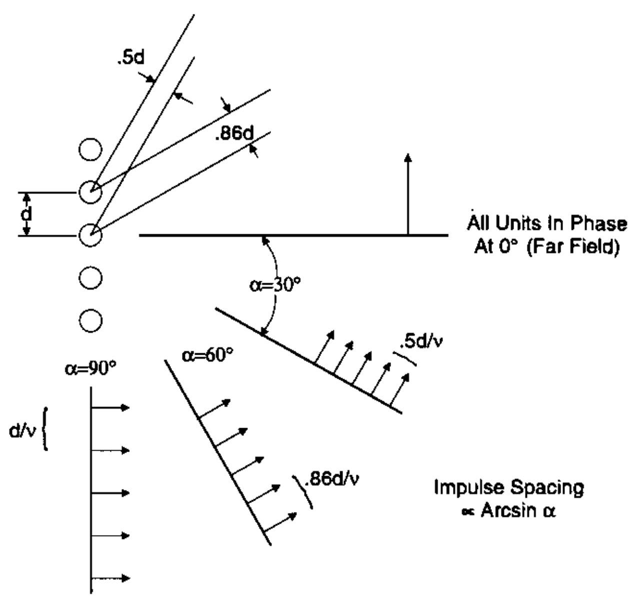

Fig. 16. Pulse train compresses toward normal axis.

and for 1 wavelength to fit,

$$\arcsin {}^{1}/_{3} = 19.47° \,.$$

These angles compare well with the angles of the lobes of Fig. 18.

For the general case, where an arbitrary number of wavelengths is found within the element spacing, the formula is

$$\text{lobe angle} = \arcsin nv/df$$

where

n = all integers from 1 to d/λ
v = speed of sound, = 344 m/s
d = element-to-element spacing, meters
f = frequency of polar curve.

It can be seen that the arcsin function allows us to map the DFT of our pulse train into a far-field polar response. In effect, the lobes and their repetitions can be thought of as spatial aliasing of the DFT wherever element-to-element spacing is greater than one-half the wavelength of interest. Clearly our lobing is a consequence of our failure to observe the Nyquist criterion.

2.2 Weighted Coefficients and the Magic of the Bessel Array

So it seems that for any array with element spacing greater than ½ wavelength we are committed to having a polar response with inherent lobing. Recent papers, though, have talked of Bessel arrays having polar responses with multiple elements that are unchanged from the polar response of a single element [3]. Doesn't this violate our understanding of the cause of lobing?

The five-element Bessel array is defined as having the following coefficient sequence:

$$+0.5 \quad -1 \quad +1 \quad +1 \quad +0.5\,.$$

The polar program was written such that each element could be given any positive or negative real value. This real value becomes the magnitude of the vector representing that element.

Fig. 19 shows the far-field polar response of a five-element array with Bessel weighting. As described, the polar response is essentially circular with minor ripples. Apparently the Bessel array does circumvent our understanding that polar lobing must come with greater than ½ wavelength element spacing. It was hoped that the Fourier transform of the Bessel sequence would shed some light on this apparent inconsistency.

The DFT of the Bessel sequence is plotted in Fig. 20 and can be compared to the plot of a 5 × 1.0 or unweighted sequence in Fig. 17. Notice that while the unweighted sequence has maximum energy at low frequencies, its energy reduces significantly by the Nyquist frequency. Thus in the aliased view (showing the response to several times the sampling frequency), it would show a cycling or "lobing." The Bessel array, though, has a unique property of essentially constant output from dc through the Nyquist frequency. No polar lobing is apparent because energy does not fall with rising frequency nor with the frequency's polar equiva-

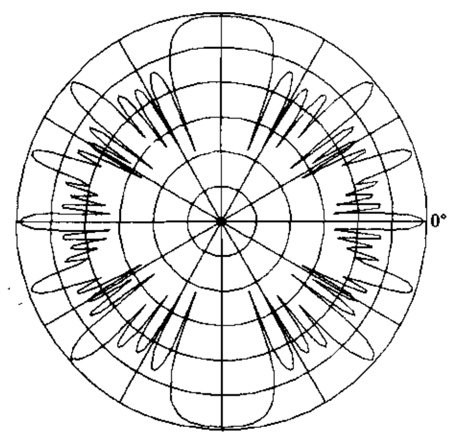

Fig. 18. Far-field polar response of five unweighted elements.

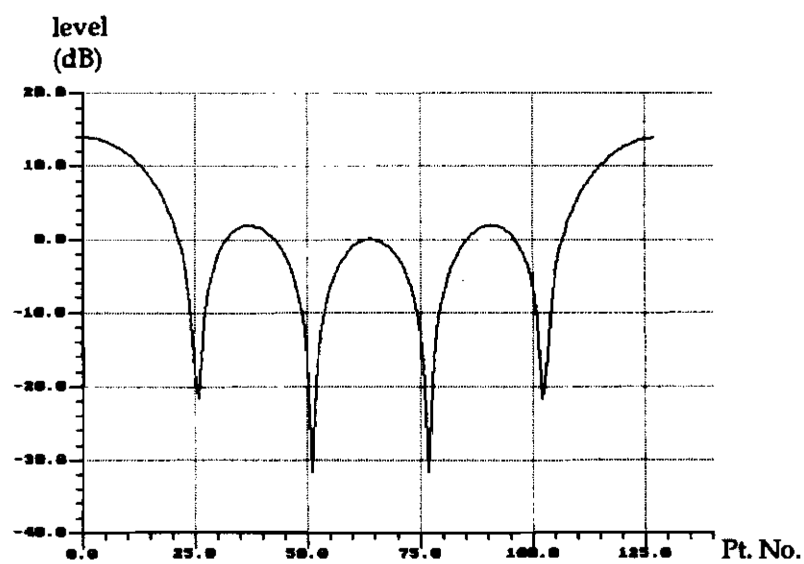

Fig. 17. 128-point Fourier transform of five-impulse sequence.

lent of changing angle. The slight ripple seen in the Fourier transform is of course present in the polar response and is mapped via the arcsin function into the spans between the (nonexistent) "lobing angles."

3 TOWARD THE OPTIMUM LINE ARRAY

Much of the literature on line arrays covers the use of "tapering" schemes, whereby electrical or mechanical means are used to vary the frequency response or level of each element. Klepper and Steele [4] cite a University column loudspeaker with full-range central elements (coaxials) combined with limited-bandwidth outer units. Their own design features a 13-unit column with fiberglass wedges that are progressively thicker near the ends to provide a continuous frequency rolloff for elements away from the center of the array. More current papers use digital filtering and computer control for a more sophisticated means to the same end [5].

Augspurger [6] makes the distinction between frequency-independent tapering, which he calls shading, and frequency-dependent tapering (modifying the frequency response element by element). This new-found understanding of the relationship between Fourier transform and polar response suggests a level tapering means of achieving a smoother polar response: *The inverse Fourier transform of the desired polar curve defines the weighting function of the required array.*

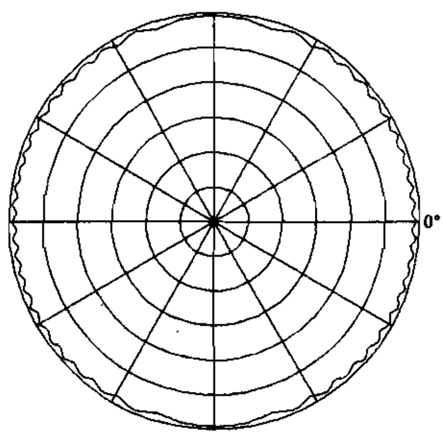

Fig. 19. Far-field polar response of five-element Bessel array.

References on line arrays often include polar responses based on the function sin x/x. For example, Olson and Schaudinischky et al. show that the polar form of a continuous line source follows a sin(sin a)/sin a pattern [1], [7]. With the understanding of the interdependence of the polar response with the Fourier transform of array weighting functions, this is understandable. The Fourier transform of a rectangular pulse has the shape of sin x/x. The DFT of a sampled pulse train of impulses of equal magnitude (which we will later refer to as rectangular weighting) would have the same form. So the polar response of a continuous line source or a line source made up of equally driven elements should always have a sin x/x form. Since the Fourier transform is reversible, the transform of a sin x/x function is rectangular in shape (a fact used to create "square" linear-phase CD oversampling filters). We should be able to create an array with sin x/x weighting that has a "rectangular" polar response. A rectangle plotted in polar form would be a "pie"-shaped wedge with no level change across the top and steep side drop-off.

Fig. 21 shows the coefficients of just such an array. Twenty-three tweeters were driven by resistive networks to approximate the center lobe and the first two sidelobes each side of a sin x/x function. Fig. 22 shows the predicted polar response at 1 kHz. Results are impressive with a level topped 60° beamwidth and a quick drop-off with over 20-dB reduction of all side responses. Measurement of the actual array shows good agreement in frontal shape and 20-dB or better "out of band" rejection (Fig. 23). However, modeling the array at 8.6 kHz (2 wavelength element spacing) shows the expected beam reduction and emergence of sidelobes (Fig. 24).

Constant directivity is possible for such an array. It would require that the sin x/x weighting scheme would expand or contract in direct proportion to the applied

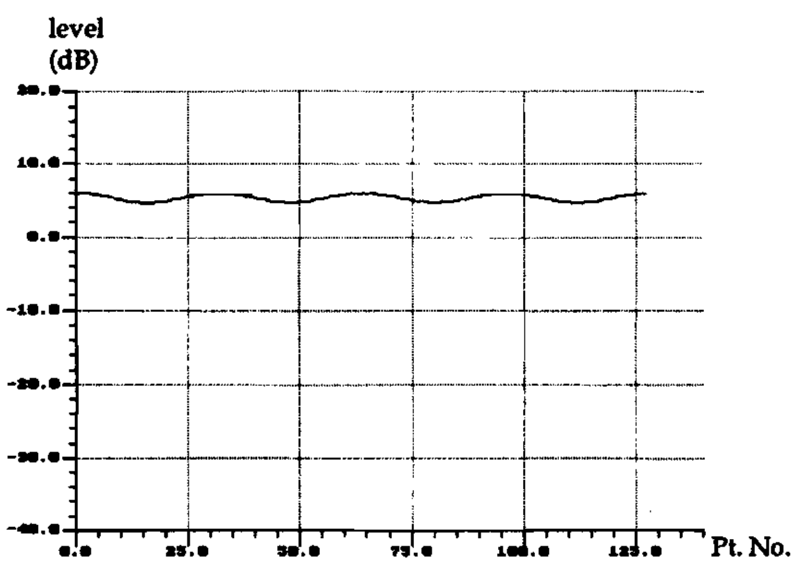

Fig. 20. 128-point Fourier transform of five-element Bessel sequence.

wavelength. For example, by 2 kHz (the octave above 1 kHz) the weighting function could contract to give the same profile over 11 tweeters, not 23. Digital methods would be ideal for achieving this contracting weighting function [5], [8]. Sidelobe creation can be forestalled by greater element density, at least for the center of the array, but not easily, and not at low cost.

4 NEAR-FIELD COMPUTER MODEL

So far we have explored the nature of the polar response of arrays at low frequencies through the region where sidelobes form. We have explained the cause of lobing and used the Fourier transform to alter and improve the shape of the forward lobe. Yet we still have poor correlation of the initial predictions and the corresponding measurements for the higher frequency range. Although the multiple lobes are observed to form and fold inward with increasing frequency, the extreme sharpness of these lobes, predicted by the model, is never observed. The actual high-frequency broadening of the measured polar curve is still unexplained. The perception that "arrays have uniform responses when observed from within their endpoints" is not substantiated by our polar model.

One way these remaining contradictions can be rationalized is if the near-field performance is significantly different from the far-field performance. Recall that the initial observation was that arrays were roughly uniform in output within their endpoints *at typical listening distances*. To explore the effects of observation distance, it was felt that a model with near-field considerations would be a better one.

The far-field polar model assumed that all paths to the observer are parallel. It also assumed that an individual element's distance differences due to the array's rotation impact the arrival phase, but not the level. Indeed, from a significant observation distance this is true. Level variation requires a significant *percentage* variation in distance. This percentage distance change due to rotation causing some elements to come nearer the observer (and some farther) is made insignificant as the observation distance becomes large.

Tweeter Number	Strength	
1	.0818	
2	.1273	
3	.1000	
4	0	
5	-.1286	
6	-.2122	
7	-.1801	
8	0	
9	.3001	
10	.6366	
11	.9003	
12	1.0000	
13	.9003	
14	.6366	
15	.3001	
16	0	
17	-.1801	
18	-.2122	
19	-.1286	
20	0	
21	.1000	
22	.1273	
23	.0818	

Fig. 21. Coefficients for 23-element sin *x*/*x* array.

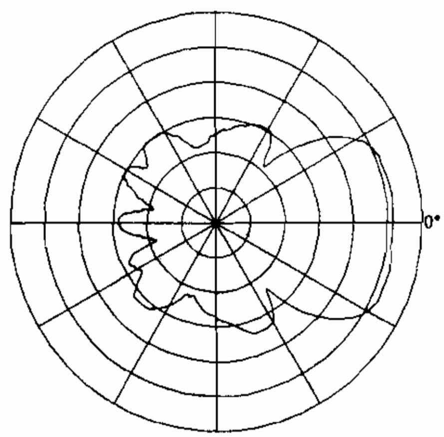

Fig. 23. Measured polar response of 23-tweeter sin *x*/*x* array, 1 kHz.

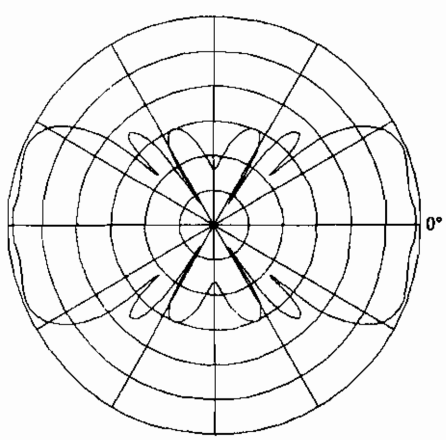

Fig. 22. Calculated far-field polar response of sin *x*/*x* array, 1 kHz.

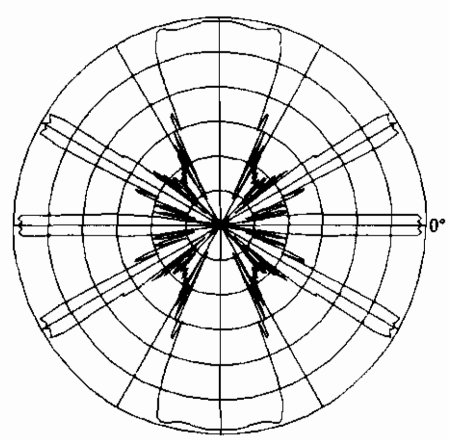

Fig. 24. Calculated polar response of sin *x*/*x* array at 8.6 kHz.