The effect of amplifier output impedance (and "damping factor") on the behavior of loudspeakers

- MrPeabody's Mom

It is a forgone conclusion that the great majority of modern amplifiers have output impedance below the threshold where it would affect the loudspeaker's performance. But this is not likely true for *all* amplifiers currently in production, and even if it *is* true for all amplifiers currently in production, output impedance is still important because it *will* affect loudspeaker performance if it is too great. Note that I'm talking about the actual output impedance, not the damping factor. The damping factor is a related specification that uses a dummy value for the speaker impedance. Damping factor would be practically useless were it not for the fact that in effect it specifies the output impedance. To calculate the output impedance, divide the damping factor by whatever dummy value the manufacturer used for the speaker impedance.

There are two distinct means by which an amplifier's output impedance can potentially affect the speaker's frequency response. One of them is only remotely related to damping, but since it is the more important of the two, I'll discuss it first. It has to do with the fact that the voltage split between the speaker impedance and the amplifier's output impedance will be frequency-dependent if speaker impedance is frequency-dependent. Each impedance's proportional share of the amplifier's true output voltage is the same as its proportional share of the total series impedance. In the impedance curve for a typical speaker, the dips are not much lower than the nominal value, whereas the peaks are tall and skinny. At the peaks, the speaker's share of the true amplifier output voltage can potentially be much greater than its nominal share, i.e., much greater than the mean value of its share over the full audio spectrum. We need to define a few variables:

OI : the amplifier Output Impedance
NI : the Nominal Impedance of the speaker (i.e., the mean value over the full audio spectrum)
PI : the Peak Impedance of the speaker

Note that if you are concerned with whether the resistance of the speaker wire is significant, you can calculate or measure this resistance and include it in the amplifier output impedance, or else subtract this resistance from the allowed output impedance after you've calculated it.

The speaker's nominal share of the amplifier output voltage is given by this expression:

$$\text{nominal voltage} = \text{true amplifier output voltage} \times NI / (NI + OI)$$

The speaker's share of the amplifier output voltage at an impedance peak is:

$$\text{peak voltage} = \text{true amplifier output voltage} \times PI / (PI + OI)$$

We want to express the ratio of these two voltages in decibels. Let's call this "Peak_dB".

$$\text{Peak\_dB} = 20 \text{ Log} [\text{ peak voltage / nominal voltage }]$$

$$\text{Peak\_dB} = 20 \text{ Log} [PI/(PI + OI) \; / \; NI/(NI + OI)]$$

$$\text{Peak\_dB} = 20 \text{ Log} [PI(NI + OI) / NI(PI + OI)]$$

In general the goal is to insure that Peak_dB will be under some specific, chosen limit, most likely in the ballpark of .1 to 1, depending on the threshold at which a response peak is deemed audible. The variable Limit_dB is another decibel value. No one enjoys solving inequalities. It's a dirty job, but since Mike Rowe isn't gonna do it:

$Peak\_dB < Limit\_dB \Rightarrow$

$20 \text{ Log } [PI(NI + OI) / NI(PI + OI)] < Limit\_dB$

$\text{Log } [PI(NI + OI) / NI(PI + OI)] < Limit\_dB/20$

$PI(NI + OI) / NI(PI + OI) < 10\text{^}(Limit\_dB/20)$

Note that the inequality immediately above was obtained by submitting both sides of the previous inequality to the exponential function '10^X'. The reason this is legit is that the exponential function increases continuously from -infinity to +infinity. As such, for any N and M, the inequality '10^N < 10^M' is true *if and only if* the inequality 'N < M' is true. The values of N and M for which one of these inequalities is true are the very same values for which the other inequality is true. Now to work on that rational exponent:

$PI(NI + OI) / NI(PI + OI) < (10\text{^}(1/20))\text{^}Limit\_dB$

$PI(NI + OI) / NI(PI + OI) < (10\text{^}.05)\text{^}Limit\_dB$

$PI(NI + OI) / NI(PI + OI) < 1.122\text{^}Limit\_dB$

$PI(NI + OI) < 1.122\text{^}Limit\_dB \times (NI \times PI + NI \times OI)$

$PI \times NI + PI \times OI < 1.122\text{^}Limit\_dB \times NI \times PI + 1.122\text{^}Limit\_dB \times NI \times OI$

Subtract 'PI x NI' from both sides and subtract '1.122^Limit_dB x NI x OI' from both sides:

$PI \times OI - 1.122\text{^}Limit\_dB \times NI \times OI < 1.122\text{^}Limit\_dB \times NI \times PI - PI \times NI$

$OI \times (PI - 1.122\text{^}Limit\_dB \times NI) < PI \times NI \times (1.122\text{^}Limit\_dB - 1)$

(I) $\quad \mathbf{OI < PI \times NI \times (1.122\text{^}Limit\_dB - 1) / (PI - 1.122\text{^}Limit\_dB \times NI)}$

In the last step above, both sides of the inequality were divided by '(PI - 1.122^Limit_dB x NI)'. The direction of the inequality has to be reversed if this expression is < 0. In general when this situation occurs when solving an inequality it is necessary to pursue two solutions. However it is apparent that if this expression is < 0, the right side of **(I)** will be < 0, which means that after reversing the direction of the inequality we would only be requiring OI to be greater than some negative value, and doing so for PI values that are just barely greater than NI (i.e., for PI < NI x 1.122^Limit_dB). This other solution is thus a superfluous solution that we may simply ignore.

To do a sanity check on the solution, we need to choose values for NI, PI and Limit_dB, then evaluate the right-hand side of **(I)** to see how great OI can be such that the response peak will be within the limit we choose. Following this we need to choose values for OI just slightly below and slightly above the threshold, to confirm that Limit_dB will be exceeded for the larger value of OI but not for the smaller value. Let's use some realistic values:

NI = 6 ohms; PI = 30 Ohms; Limit_dB = .5 dB

$OI < PI \times NI \times (1.122\text{^}Limit\_dB - 1) / (PI - 1.122\text{^}Limit\_dB \times NI)$

$OI < 30 \times 6 \times (1.122\text{^}.5 - 1) / (30 - 1.122\text{^}.5 \times 6)$

$OI < .451 \text{ ohms}$

This seems a reasonable result, so let's calculate the response peak for two values of OI, one value slightly below .451 ohms and the other value slightly above .451 ohms. Let's use .450 ohms and .452 ohms.

Peak = 20 Log [PI/(PI + OI) / NI/(NI + OI)]

Peak = 20 Log [30/(30 + .45) / 6/(6 + .45)]

Peak = 20 Log [1.0591133] = .499 dB

When OI is slightly less than the threshold obtained via **(I)**, the response peak is very slightly less than the chosen limit of .5 dB. Let's see what happens when OI is equal to .452 ohms.

Peak = 20 Log [30/(30 + .452) / 6/(6 + .452)]

Peak = 20 Log [1.0593721] = .501 dB

The inequality **(I)** thus appears to be correct.

How much lower will OI need to be if we use .1 dB for Limit_dB?

OI < 30 x 6 x (1.122^.1 -1) / (30 - 1.122^.1 x 6)

OI < .0871 ohms

It is interesting to observe that in both of these realistic cases, with Limit_dB assigned values of .5 dB and .1 dB, the calculated value for OI in ohms is approximately 90% of Limit_dB. This suggests a useful first-order approximation for estimating the maximum allowable amplifier output impedance in ohms. However the two cases are both specific to a speaker with 6 ohm nominal impedance. If we repeat the calculations for 4 ohm speakers and for 8 ohm speakers we would likely come up with different approximations, and the approximations would be different again for different values for PI, the impedance peak of the speaker.

How long is my 16 gauge speaker cable allowed to be if the amplifier output impedance is .15 ohms and I use .5 dB for Limit_db (with the same values for NI and PI)?

.451 ohms -.15 ohms = .301 ohms

The resistance of 16 gauge wire is about 4 ohms per thousand feet (at room temperature):

.301 ohms x 1000 ft / 4 ohms = 75 feet.

Since the true distance is double the length of the two-wire cable, you have to divide this in half. As long as each cable is no greater than 35 feet, it does not matter if they are not equal.

The same inequality works for the impedance dips, and allows us to get an accurate sense of the extent to which the impedance dips may pose tighter constraints for the speaker cable. If the nominal impedance is 6 ohms, the minimum impedance (this is the voice coil's DC resistance) will be maybe 4.5 ohms. All we need to do is redo the calculation using 4.5 ohms for PI and with Limit_dB set to -.5 dB instead of +.5 dB.

OI < PI x NI x (1.122^Limit_dB -1) / (PI - 1.122^Limit_dB x NI)

OI < 4.5 x 6 x (1.122^-.5 -1) / (4.5 - 1.122^-.5 x 6)

OI < 1.3 ohms

It may seem as though the impedance dips would impose tighter constraints on the speaker cable, in order that the cable resistance will be adequately small in relation to the speaker impedance. As concerns the affect on the speaker's frequency response it is only the impedance peaks that matter, and the nominal speaker impedance suffices for consideration of the overall loss in the cable.

As for the other means (the means that actually does have something to do with damping), the best way to approach it is probably from the perspective of speaker Q. For both sealed enclosures and enclosures that use a port or a passive radiator, the overall shape of the response is largely determined by the system Q. System Q is linearly dependent on driver Q. If driver Q is increased by some specific percentage and this is not compensated by making the enclosure larger, the percentage increase in system Q will be the same as the percentage increase in driver Q.

The electrical damping of the driver makes up the major share of the driver's damping and thus has a strong hand in determining the driver Q. The equations that tell us precisely how driver Q depends on the resistance in the voice coil circuit (primarily the coil itself) tell us that driver Q has a quasi-linear dependence on the total resistance in the circuit. Thus, when the amplifier's output impedance is added to the voice coil resistance, the system Q increases by roughly the same percentage as the increase in the resistance in the voice coil circuit. This assumes that the enclosure volume is not increased to compensate; this is a good assumption given that it is not practical to accommodate differences in amplifiers by modifying the speaker enclosure. Also worth noting is that this relationship would be exact except for the fact that the driver Q is also influenced by mechanical damping, which is typically very small in comparison to the electrical damping. With the great majority of drivers, the total Q is not very different from the electrical Q, and the total Q of the driver has a sort of quasi-linear dependence on the total resistance in the voice coil circuit.

Thus, if we desire for the percentage increase in system Q to remain under some limit L, we need to insure that the amplifier's output impedance (the real part of it) is no greater than L% of the voice coil's DC resistance. For example, if the DC resistance of the coil is 4.5 ohms and we want to keep the increase in system Q to less than, say, 10%, this means that the amplifier's output impedance will need to be less than (approximately) .45 ohms. Reportedly, most all modern speaker amplifiers meet this requirement with ample room to spare. Some of the economy class D amplifiers may be an exception, however the output impedance of even these amplifiers will likely be very low at the low frequency where driver damping is needed toward keeping the enclosure small. Some of the tube amplifiers may similarly be an exception.

All in all the reasonable conclusion seems to be that in a very small but uncertain percentage of cases, a speaker amplifier's output impedance may be just barely great enough to produce a barely perceptible increase in speaker Q. If you use a tube amp or a class D amp, or if you use a class D headphone amplifier with a pair of highly sensitive headphones (they'll likely have low impedance), or if you plug a typical pair of headphones into the headphone jack on most any receiver or integrated amplifier intended primarily for speakers, it is not out of the question that the frequency response of your speakers or headphones will be very slightly affected by the amplifier's output impedance, via one or the other of the two means, possibly both of them acting in unison.